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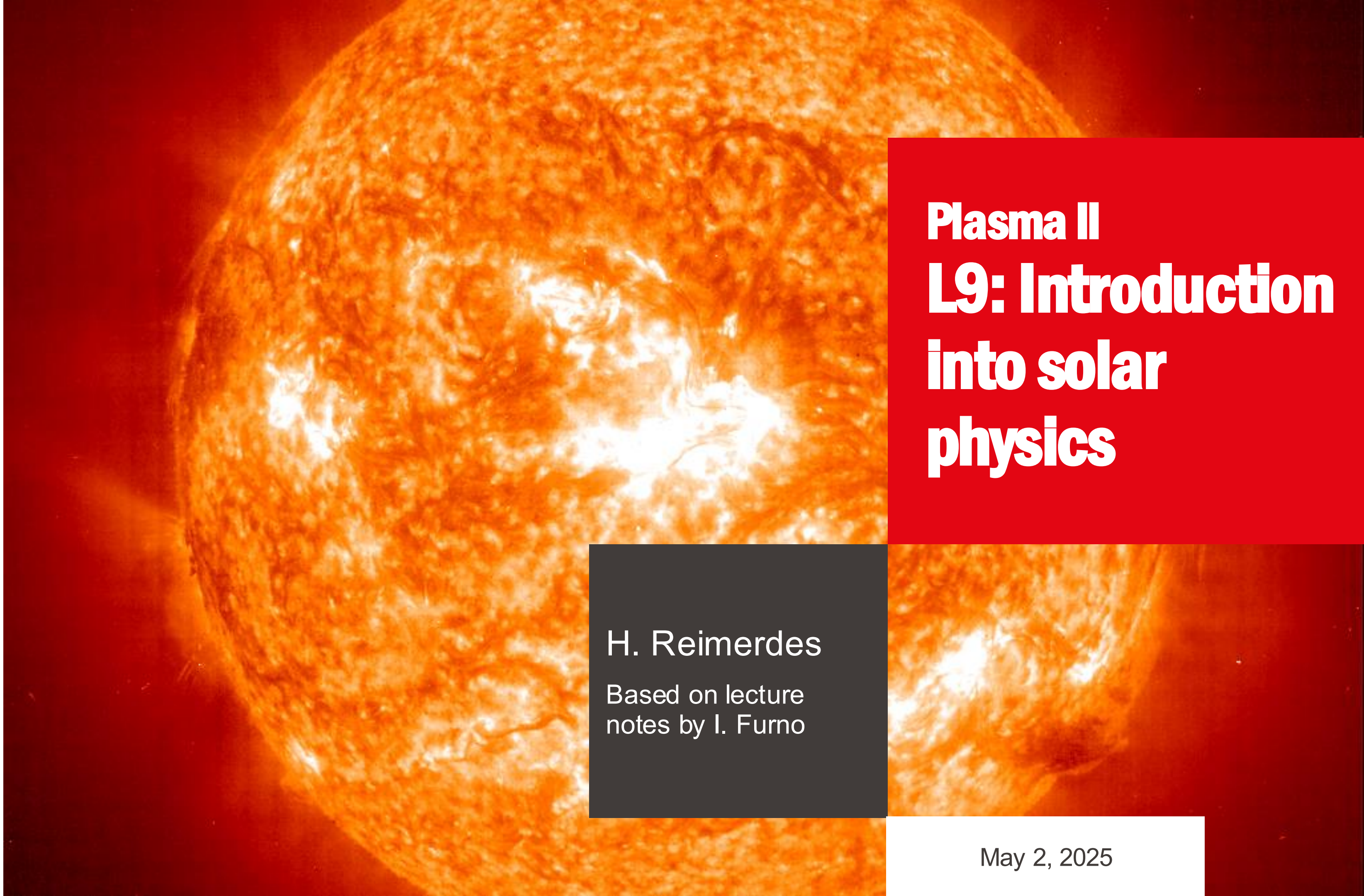

Plasma II
L9: Introduction into solar physics

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Based on lecture notes by I. Furno

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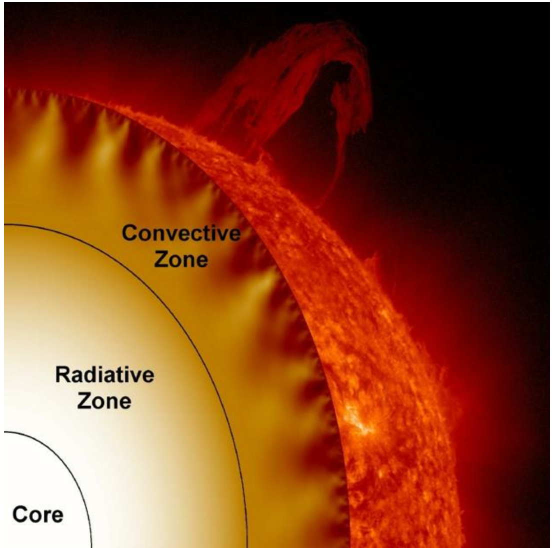
Swiss Plasma Center

Content of astrophysics module



- The sun's nuclear energy source
- Transport processes

} L9

- The structure of its magnetic field
- The solar dynamo

} L10

- Magnetic reconnection
- The heliosphere
- Solar wind

} L11

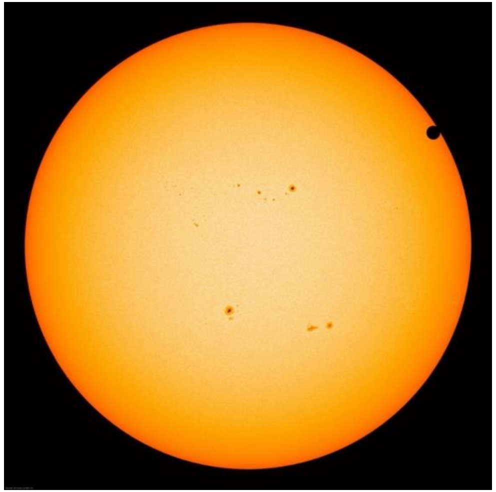
➢ See also EPFL MOOC "Plasma physics: Applications" #4a-b
https://learning.edx.org/course/course-v1:EPFLx+PlasmaApplicationX+1T_2018/home

- N. Meyer-Vernet, "Basics of the solar wind", Cambridge Atmospheric and Space Science Series, Section 3

A few properties of our Sun

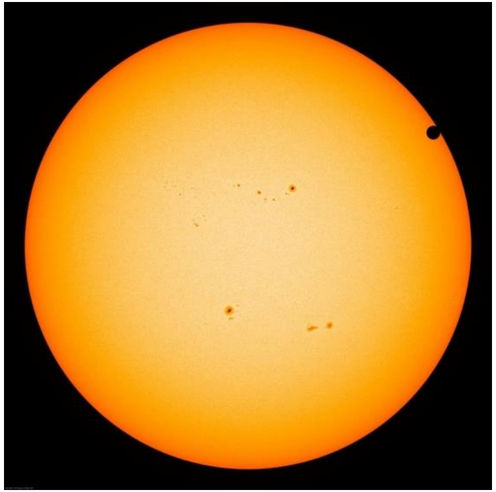
Mean distance to Earth	$d_{\oplus}$ **= 1.5 x 10^{11} m**
Radius	$R_{\odot}$ **= 7.0 x 10^{8} m**
Mass	$M_{\odot}$ **= 2.0 x 10^{30} kg**
Luminosity	$L_{\odot}$ **= 3.84 x 10^{26} W**

- Solar distance $\mathbf{d_{\oplus}}$ is called astronomical unit (AU) and is a basic unit in the Solar System and beyond



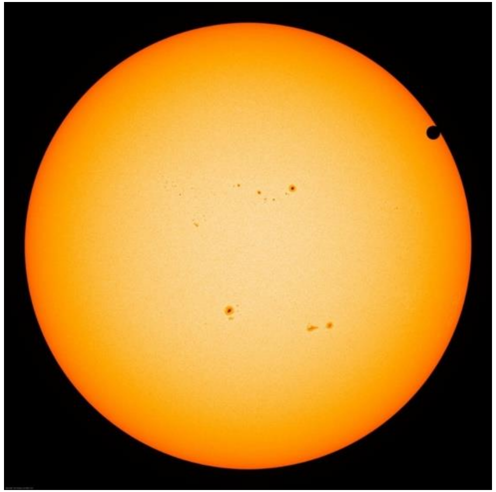
A few properties of our Sun

- Sun's radius ~half of the visible disc: 0.5° as seen from Earth
  - Almost perfectly round and sharply defined ← virtually all the light originates from a thin surface layer ("photosphere")
  - The photosphere has a temperature of ~5500K (blackbody estimate)



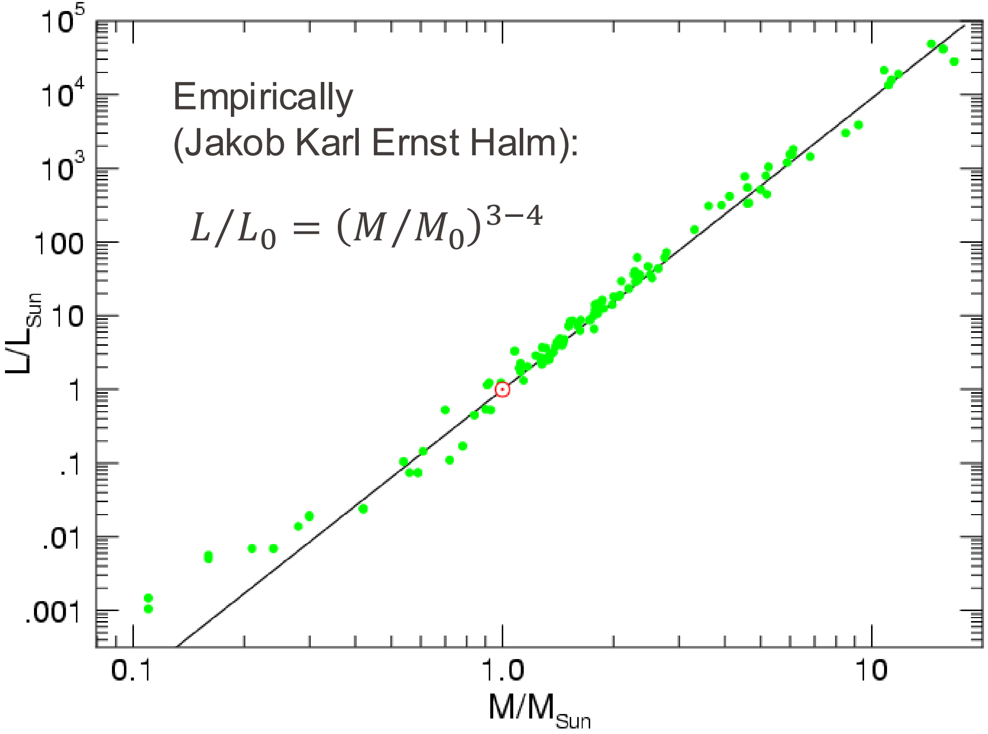
A few properties of our Sun

- Elemental abundances are typical for cosmic sources, but different from Earth
  - 73.4%* Hydrogen
  - 25.0%* Helium
  - 1.6%* everything else (“metals”)



*by mass

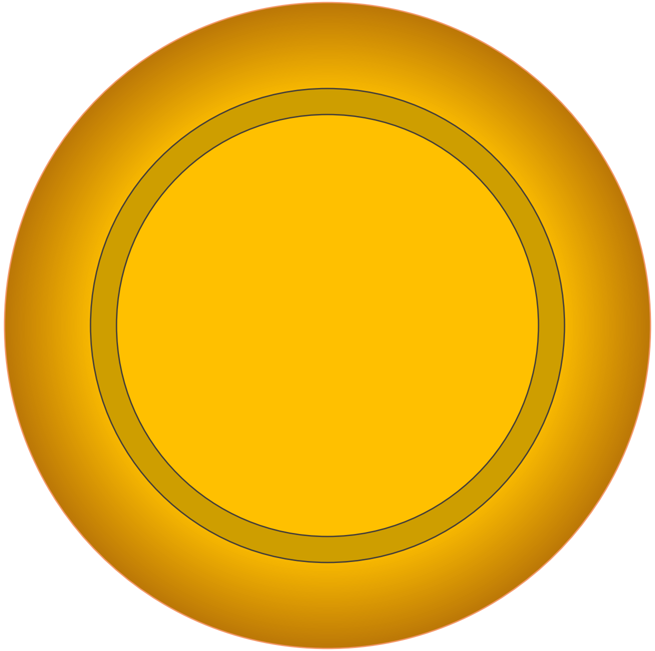
Luminosity-mass dependence for main sequence stars



Lifetime of a star

- Lifetime of stars decreases with increasing mass

Hydrostatic equilibrium in spherical geometry



- **Static equilibrium**: Pressure forces balance gravitational forces

$$\frac{dP}{dr} = -\frac{\rho M_r G}{r^2}$$

Hydrostatic equilibrium in spherical geometry

- **Assumption**: Sun is in a *hydrostatic equilibrium*

$$dP/dr = -\rho M_r G/r^2$$

where M_r is the mass inside the radius r

- Multiplying both sides by $4\pi r^3$ and integrating the L.H.S. by parts between r = 0 and $r = R$ (where the pressure is negligible)

$$\underbrace{-3\int_0^R P\,4\pi r^2 dr}_{-3\langle P\rangle V \,=\, -6Nk_B\langle T\rangle} = \underbrace{-\int_0^R \rho\,(M_r G/r^2)\,4\pi r^3 dr}_{\text{Gravitational energy of the sun}}$$

$-3\langle P\rangle V = \boldsymbol{-6Nk_B\langle T\rangle}$ Gravitational energy of the sun

- **Assumption**: the volume V is filled with N protons and N (free) electrons with an average temperature $\langle T\rangle$

$$\langle P\rangle \approx 2Nk_\mathrm{B}\langle T\rangle/V$$

Hydrostatic equilibrium in spherical geometry

- Hydrostatic equilibrium

$$-6Nk_{\mathrm{B}}\langle T\rangle = -\int_0^R \rho\,(M_r G/r)\,4\pi r^2 dr$$

- Assume constant density ρ and integrate after substituting $M_r = \rho 4\pi r^3/3$

$$E_g = -\rho^2 G (4\pi)^2 R^5/15$$

- Substitute $\rho = m_p N/V$ with $V = 4\pi R^3/3$ to obtain

$$E_g = -\frac{3}{5}\frac{m_p^2 N^2 G}{R}$$

Assumption of a hydrostatic equilibrium yields temperature estimate

- Hydrostatic equilibrium

$$-6Nk_B\langle T\rangle = -\frac{3}{5}\frac{m_p^2N^2G}{R}$$

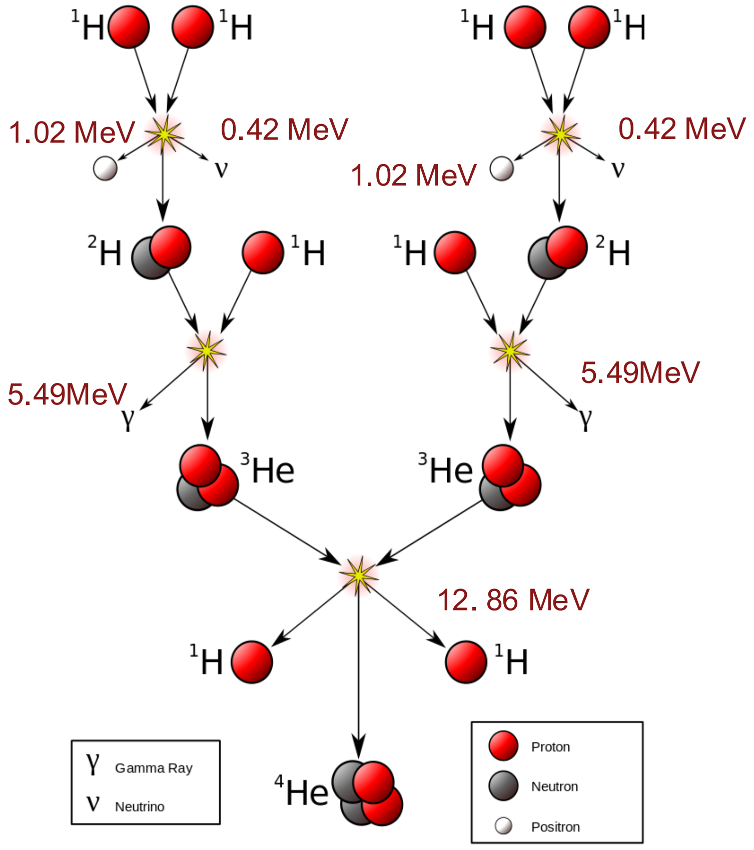
$$\Leftrightarrow \qquad \langle T\rangle = \frac{m_p^2G}{10k_B}\frac{N}{R}$$

- An estimate of the total number of protons $N_\odot = M_\odot/m_p \approx 1.2\times 10^{57}$ and the measured radius $R_\odot \approx 7\times 10^8$ m allows to estimate the Sun's average temperature

$\langle T\rangle \approx 2.3\times 10^6 K$ → **Plasma!**

  - Central temperature significantly larger!

The proton-proton chain reaction



- At a core temperature of $\gtrsim 10^7$K, the main fusion reaction is the p-p I chain

- How long the Sun will last by radiating its luminosity?

Summary

- Approximate sun as a spherical hydrogen plasma in hydrostatic-equilibrium
- Estimate temperature from its mass and radius → Sun’s temperature sufficient for p-p fusion

The luminosity of the Sun

- A hot body in local thermal equilibrium (LTE) at a temperature T contains photons of energy density

$$w_{\mathrm{ph}} = \frac{4\sigma_s}{c} T^4$$

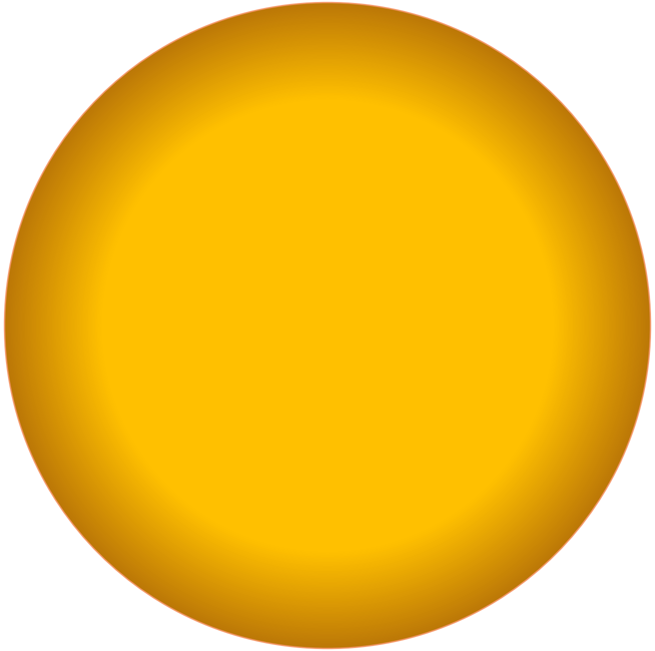
- The total radiative energy can be expressed as

$$W_{\mathrm{ph}} = \frac{4\sigma_s}{c} T^4 V$$

- The corresponding luminosity is

$$L = \frac{W_{\mathrm{ph}}}{\text{mean time } t_{\mathrm{ph}} \text{ to reach the surface}}$$

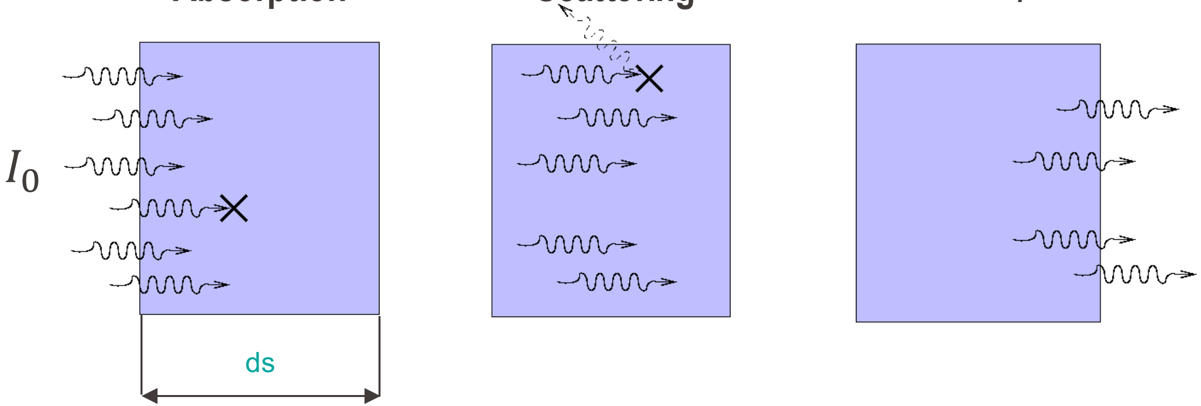
Are photons free to escape?



Opacity of a medium (plasma)

Absorption | **Scattering**

$$dI = -\kappa\rho I ds$$



$$I = I_0 e^{-\kappa\rho s}$$

Opacity κ [m^2/kg]

$$I = I_0 e^{-\tau}$$

Optical depth τ

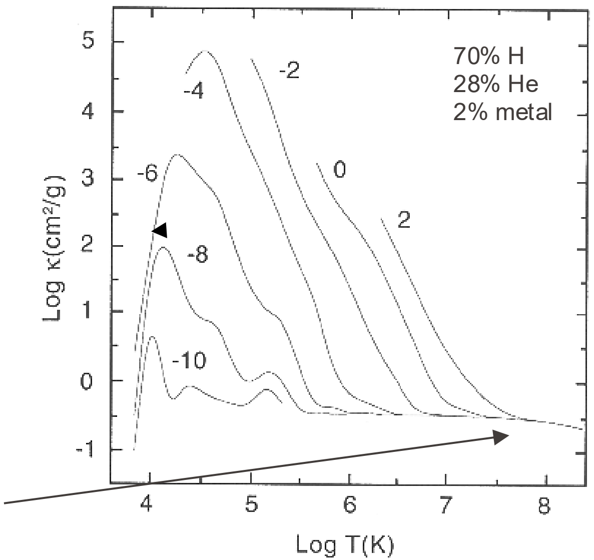
Optically thin (τ<<1) → **transparent**

Optically thick (τ >>1) → **opaque**

Sources of opacity in a plasma

- Photons of different frequencies interact differently with matter → κ is a function of frequency (i.e. temperature)
- These processes are the inverse of those considered in tokamaks for computing the radiated power in a discharge
- Interactions that lead to the Sun's opacity
  - **Free-free absorption** (inverse Bremsstrahlung: $\sigma_{ff} \propto T_e^{-3/2}$)
  - **Bound-free absorption** (photo-ionization)
  - **Bound-bound absorption** (photo-excitation)
  - **e$^-$ scattering** (Thomson or Compton)

Rosseland mean opacity



Electron scattering (Thomson) provides a base level of opacity, dominant at high temperature

Photon mean-free path and escape time

- Tor T_e>10^7 K scattering on electrons (Thomson scattering) is the dominant contribution to the opacity

$$\sigma_T = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-29} \mathrm{m}^2$$

- The resulting mean-free-path for photons is

$$\lambda_{MFP} = \frac{1}{n\,\sigma_T} = \frac{1}{N\,\sigma_T} \frac{4\pi R^3}{3} \approx 2\mathrm{cm}$$

➢ Photons scatter often before reaching the surface → diffusive transport (see L5)

$$\langle R^2 \rangle = D t_{\mathrm{ph}} \quad \text{with} \quad D = \frac{\lambda_{\mathrm{MFP}}^2}{\tau_{\mathrm{c}}}$$

- Estimate for the sun $R_\odot^2 = D t_{\mathrm{ph}} = \frac{\lambda_{\mathrm{MFP}}^2}{\lambda_{\mathrm{MFP}}/c} t_{\mathrm{ph}}$ ⇨ $t_{\mathrm{ph}} = \frac{R_\odot^2}{\lambda_{\mathrm{MFP}} c}$

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Revised estimate of the luminosity of the Sun

- Estimate of the luminosity

$$L = W_{\mathrm{ph}}/t_{\mathrm{ph}}$$

- Using the estimate of the photon escape time

$$t_{\mathrm{ph}} = \frac{R_{\odot}^2}{\lambda_{\mathrm{MFP}}\, c} \sim 8 \times 10^{10}\,\mathrm{s}$$

yields $L_{\odot} \sim 3.4 \times 10^{26}\mathrm{W}$ (see slide 5) → close to the measured $3.8 \times 10^{26}\mathrm{W}$!

Main sequence stars

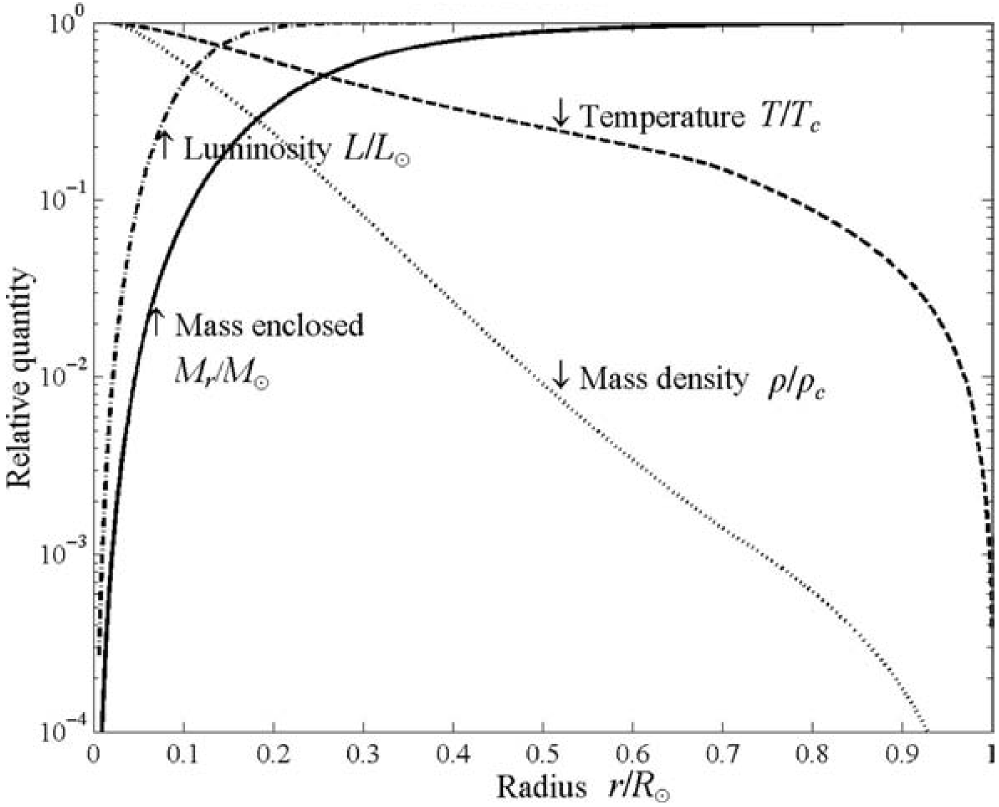
Luminosity

Summary

- Simple assumptions (sphere, ideal gas, constant density) allow to compute fundamental properties of stars
  - Average temperature
  - Luminosity, incl. mass dependence
  - Age
- (Over-) simplification
  - Contain other elements than hydrogen
  - Internal structure/profiles
  - Deviations from the ideal gas law
  - Magnetic fields

The temperature in the Sun's interior

$T_c = 1.57 \times 10^7$ K

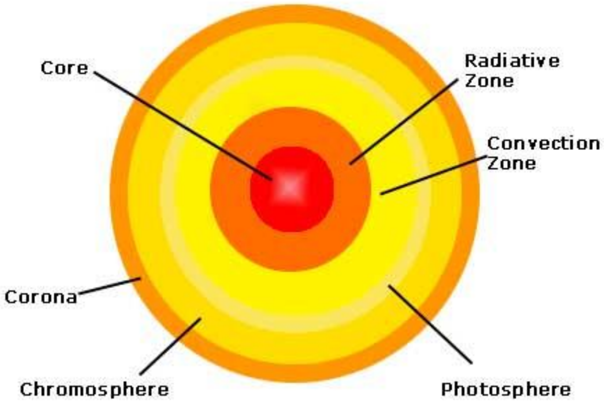


$T_s = 5.5 \times 10^3$ K

Energy transport

Energy transport in stellar interiors can occur by three mechanisms

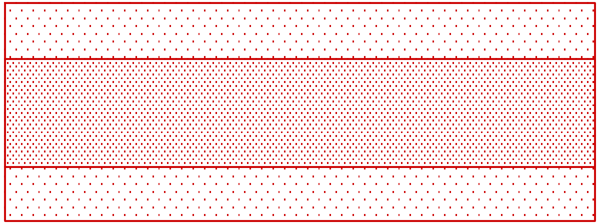
- **Radiation:** photons carry energy, but constantly interact with electrons and ions
  - Each interaction causes photons, on average, to lose energy to the plasma $\Rightarrow$ Increase in gas temperature
- **Convection:** energy is carried by macroscopic mass motion (rising gas)
  - If the (gas) density of a region is less than that of its surroundings, it rises
  - No net mass flow - rising matter is compensated by sinking matter



- **Conduction:** energy is carried by mobile electrons, which collide with ions and other electrons
  - Relevant to white dwarfs, neutron stars and also magnetised stellar atmosphere

Transport by radiation

- Consider a plasma slab of thickness dr at position r inside an isotropic atmosphere
  - Radiation pressure on the upper and lower surfaces of the slab



- The net force/unit area exerted by radiation field on slab is

$$[P_{\text{rad}}(r) - P_{\text{rad}}(r+dr)] = -\frac{dP_{\text{rad}}(r)}{dr}\,dr$$

with the radiation pressure being $P_{\text{rad}}(r) = \frac{\sigma_S T^4(r)}{3}$

- The change in radiative energy flux F_{rad} due to the opacity is $dF_{\text{rad}} = -\kappa\rho F_{\text{rad}} dr$ and therefore the net momentum transfer to the slab gas is $dP_{\text{rad}} = -\kappa\rho F_{\text{rad}} dr/c$

Transport by radiation (cont.)

- Equate transferred momentum per unit time with the net force

$$-\kappa(r)\rho(r)F(r)/c = -\frac{dP_{\mathrm{rad}}(r)}{dr} = \frac{4\sigma_s T^3(r)}{3}\frac{dT(r)}{dr}$$

$$\Rightarrow \qquad \frac{dT(r)}{dr} = -\frac{3\kappa(r)\rho(r)F(r)}{4\sigma_s c T^3(r)}$$

- With $F(r) = L(r)/(4\pi r^2)$ we obtain the *radiative transport equation*

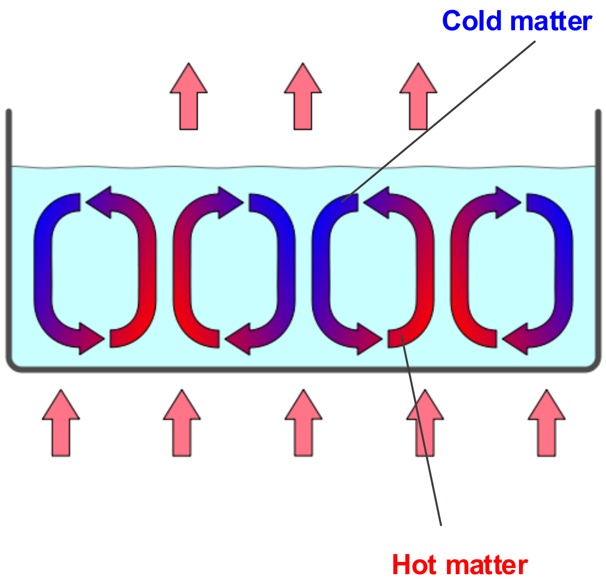
$$\frac{dT(r)}{dr} = -\frac{3\kappa(r)\rho(r)L(r)}{16\pi\sigma_s c\, r^2\, T^3(r)}$$

- **Large temperature gradients** are favored by either a **low temperature** and a **high density** and **high opacity**
  - ➢ This happens in the outer 30% of the solar radius, where the opacity becomes far greater than in the inner region, while ρ/T^3 changes less

Convective instability

- Convection is common in fluids heated from below

- Suppose a bubble (or blob) of matter is hotter than its surroundings
  - Its internal pressure P must match the exterior one
  - With $P \propto \rho T$ it expands/is lighter than its surroundings
  - Buoyance causes bubble to rise
  - Conversely, a colder bubble tends to sink



Convective instability (cont.)

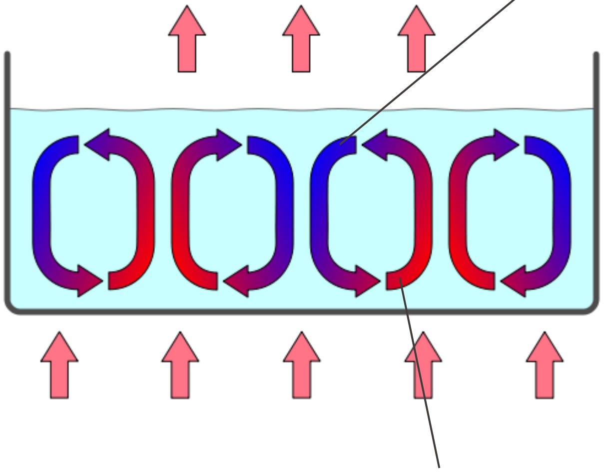
Will convection grow or stop?

- Key point: temperature and pressure decrease upward

  In rising the external pressure decreases

  Bubble expands and temperature decreases

- Temperature decrease is less than external one?
  - Yes: the bubble continues to rise
  - No: the bubble stops/sinks



Criterion for instability

- **Assumption**: the rising material expands adiabatically and changes in composition are negligible
  - With $T \propto P^{(\gamma-1)/\gamma}$ for adiabatic processes, where $\gamma = c_P/c_V$ is the ratio of the specific heats

Bubble: $$\left(\frac{dT_{\mathrm{b}}}{dr}\right)_{\mathrm{adiabatic}} = \frac{\gamma-1}{\gamma}\frac{T_{\mathrm{b}}}{P}\frac{dP}{dr}$$

- The bubble will remain buoyant and continue to rise if it remains hotter than its surroundings → its rate of temperature decrease is less then that of the surroundings (*Schwarzschild criterion*)

$$\underbrace{\left|\frac{dT}{dr}\right|}_{\text{surrounding}} > \left|\underbrace{\left(\frac{dT_{\mathrm{b}}}{dr}\right)_{\mathrm{adiabatic}}}_{\text{bubble}}\right| = \frac{\gamma-1}{\gamma}\frac{T_{\mathrm{b}}}{H}$$

What determine the pressure scale length $H \equiv \frac{P}{dP/dr}$?

Criterion for instability (cont.)

- Pressure gradient in hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho g_r = -\frac{Pm}{k_\mathrm{B}T} g_r$$

with $P = \rho k_\mathrm{B} T / m$

- ➢ Yields pressure scale length $H = \frac{k_\mathrm{B}T}{m g_r}$
- ➢ Convection can start, if

$$\left|\frac{dT}{dr}\right| > \frac{\gamma-1}{\gamma}\frac{T_\mathrm{b}}{H} \approx \frac{\gamma-1}{\gamma}\frac{m g_r}{k_\mathrm{B}} = \frac{g_r}{c_p}$$

with $c_P = \frac{\gamma}{\gamma-1}\frac{k_\mathrm{B}}{m}$

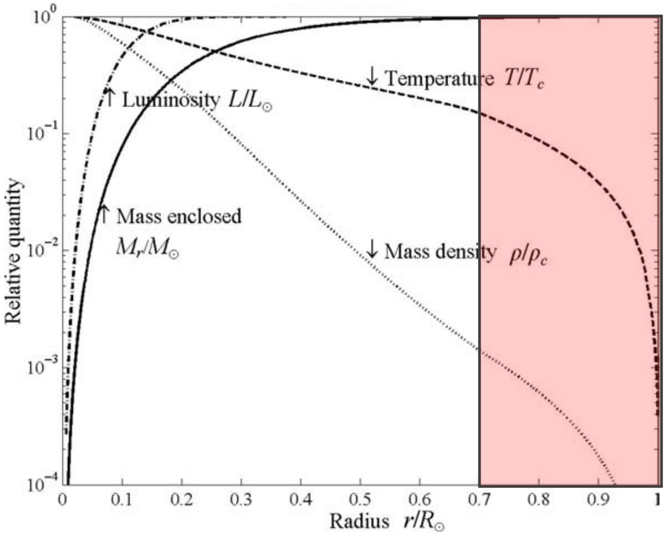
Criterion for instability (cont.)

- Convection can start, if

$$\left|\frac{dT}{dr}\right| > \frac{g_r}{c_p}$$

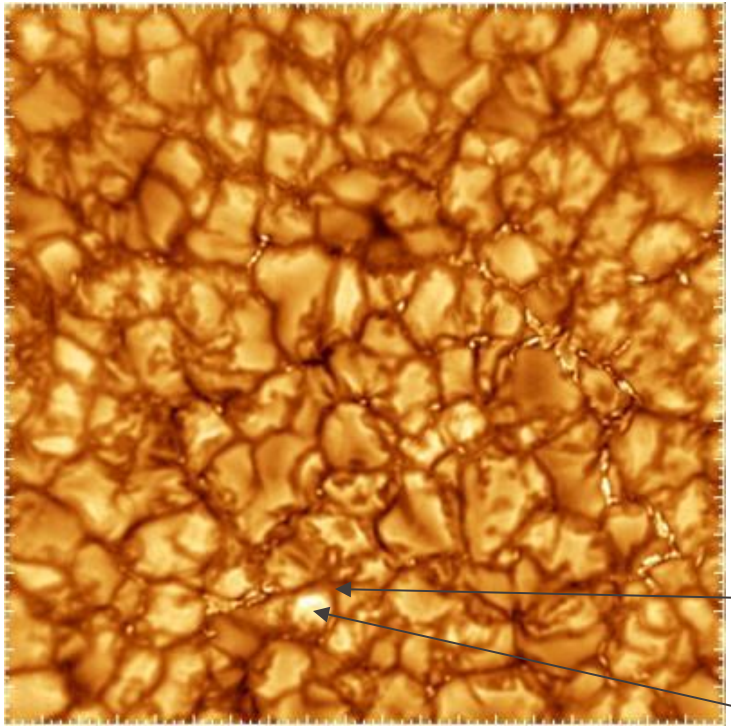
- Radiation transport leads to large temperature gradient, when opacity becomes large, i.e. when T is sufficiently low

➢ Convection dominates energy transport in the outer 30% of the sun's radius



Evidence of convection: granulation

http://www.youtube.com/watch?v=W_Scoj4HqCQ



mean size 1250 km
(all size up to ~ 2500 km)
short life time ~ 6 min

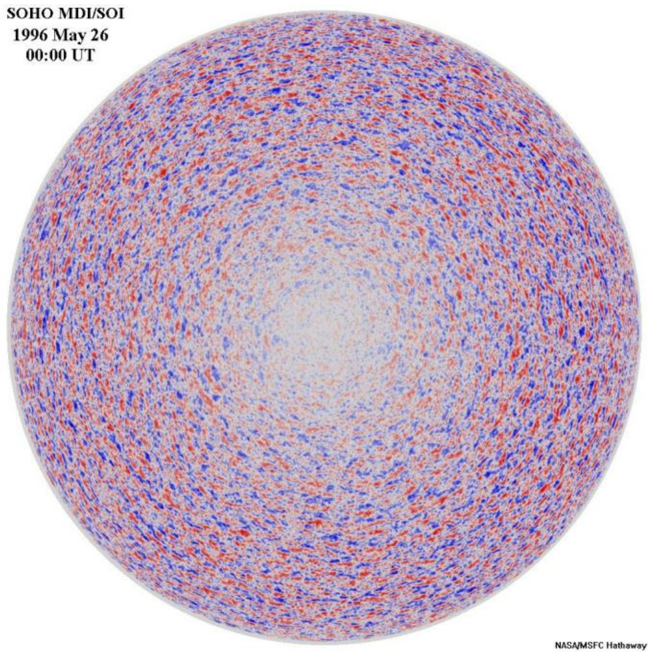
Cold matter sinking down

Hot matter raising from below

Hinode: Solar Optical Telescope

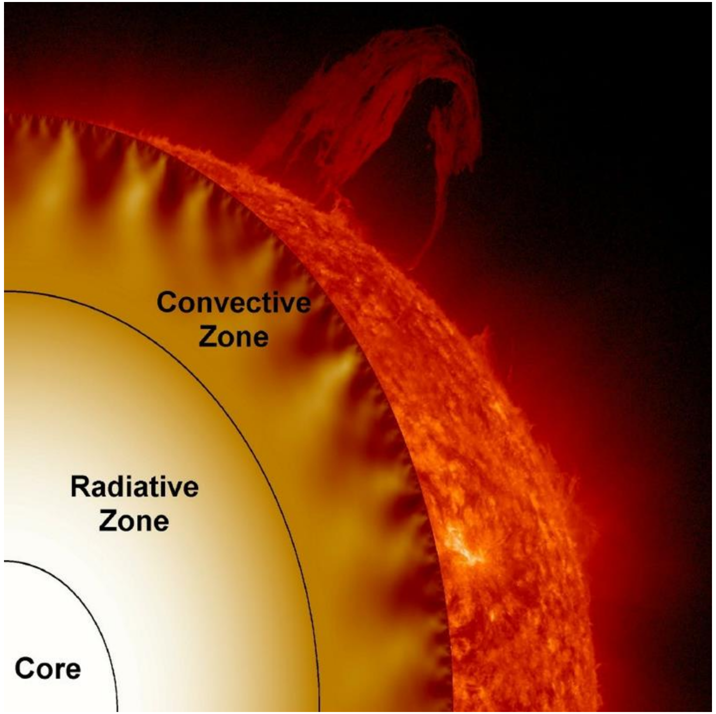
Super-granules

- Much larger versions of granules (~35,000 km across)
  - Cover the entire Sun and are continually evolving
- Best seen in measurements of the "Doppler shift"
  - Light from material moving toward us is shifted to the blue
  - Light from material moving away from us is shifted to the red
- Last for a day or two and have flow speeds of about 0.5 km/s



- Fluid flows in super-granules carry magnetic field bundles to cell edges

Summary: Transport processes



- Radiation transport dominates, where temperature is high and opacity low
  - ➢ Low T gradient at high T
- Convective transport dominates, where temperature gradient is sufficiently high
  - ➢ Transition from radiative zone to convective zone at ~0.7 of the sun's radius